Liability Design for Autonomous Vehicles and Human-Driven Vehicles: A Hierarchical Game-Theoretic Approach

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Research Questions

The tragic fatality in Arizona involving a self-driving automobile elicited tremendous attention from the public and policy makers about how to draw the lines of legal liability when AVs interact with human drivers, cyclists, and pedestrians. We aim to explore the following questions:

- ✓ How will human drivers change driving behavior when facing AVs?
- ✓ Will AVs improve road safety in a mixed AV-HV market?
- ✓ What is a social optimal policy for law maker to deal with AVs and HVs?

Literature Review

Pedersen (2001,2003)

1. established a road safety game by modelling care level as players' strategies.
2. studied "moral hazard" by investigating factors which may have effects on players' caution.

Chatterjee (2013,,2016)

1. developed a game framework by introducing share rules to model drivers' causation functions.
2. studied the effects of changing composition of HVs and AVs on driving behaviors.

Friedman, Eric (2019)

1. analyzed how varying legal standard with negligence can distort human drivers' interaction with AVs.
2. analyzed how various liability rules affect care levels of human drivers and AV manufacturers.

Shavell (2019)

argued for what is effectively a Pigouvian tax on accidents where AVs saturate the traffic ecosystem.

This work

investigates how AVs affect road safety and designs socially optimal liability rules for AVs and HVs.

Contributions

1. Building on a good understanding of both AVs and HVs equilibrium behaviors in the developed game, we explore human drivers' moral hazard incurred by the presence of AVs;
2. We aim to model how the AV manufacturer selects safety specifications for AVs as the market becomes larger. Accordingly, the role of the AV manufacturer on traffic safety is explored;
3. A sequence of sensitivity analysis is performed to investigate how the transportation system performance may change when the related parameters vary.

Hierarchical Game

Players' Disutility

$$C_{H_i}^{(HH)}(c_{H_1}^{(HH)}, c_{H_2}^{(HH)}) = \underbrace{w_h \cdot S_H(c_{H_i}^{(HH)})}_{\text{cost of executing a care level}} + \underbrace{P(c_{H_1}^{(HH)}, c_{H_2}^{(HH)}) \cdot T \cdot s_{H_i}^{(HH)}(c_{H_1}^{(HH)}, c_{H_2}^{(HH)})}_{\text{crash loss apportioned to } H_i}$$

$$C_A(c_A, c_H^{(AH)}) = \underbrace{w_a \cdot S_A(c_A)}_{\text{Sensor cost}} + \overbrace{\underbrace{p^2 \cdot P(c_A, c_A) \cdot T}_{\text{AVs' loss share in the } AA \text{ scenario}} + \underbrace{2p(1-p) \cdot P(c_A, c_H^{(AH)}) \cdot T \cdot s_A^{(AH)}(c_A, c_H^{(AH)})}_{\text{AV's loss share in the } AH \text{ scenario}}}^{\text{crash loss involved with AVs}}$$

$$SC(c_{H_1}^{(HH)}, c_{H_2}^{(HH)}, c_A(k), c_H^{(AH)}(k)) = \underbrace{w_l \cdot TC(c_{H_1}^{(HH)}, c_{H_2}^{(HH)}, c_A(k), c_H^{(AH)}(k))}_{\text{total cost of care levels}} + \underbrace{TL(c_{H_1}^{(HH)}, c_{H_2}^{(HH)}, c_A(k), c_H^{(AH)}(k))}_{\text{total crash loss}}$$

Performance Measure

Moral Hazard - a lower care level is chosen:

$$C_i^*(x) > C_i^*(x')$$

Road Safety – a lower crash rate exists in a pure AV market:

$$TR^{mixed}\left(c_{H_1}^{*(HH)}, c_{H_2}^{*(HH)}, c_A^*, c_H^{*(AH)}\right) > TR^{pure\ AV}(c_A^*)$$

Social Welfare – a lower social cost exists in a pure AV market:

$$SC^{mixed}\left(c_{H_1}^{*(HH)}, c_{H_2}^{*(HH)}, c_A^*, c_H^{*(AH)}\right) > SC^{pure\ AV}(c_A^*)$$

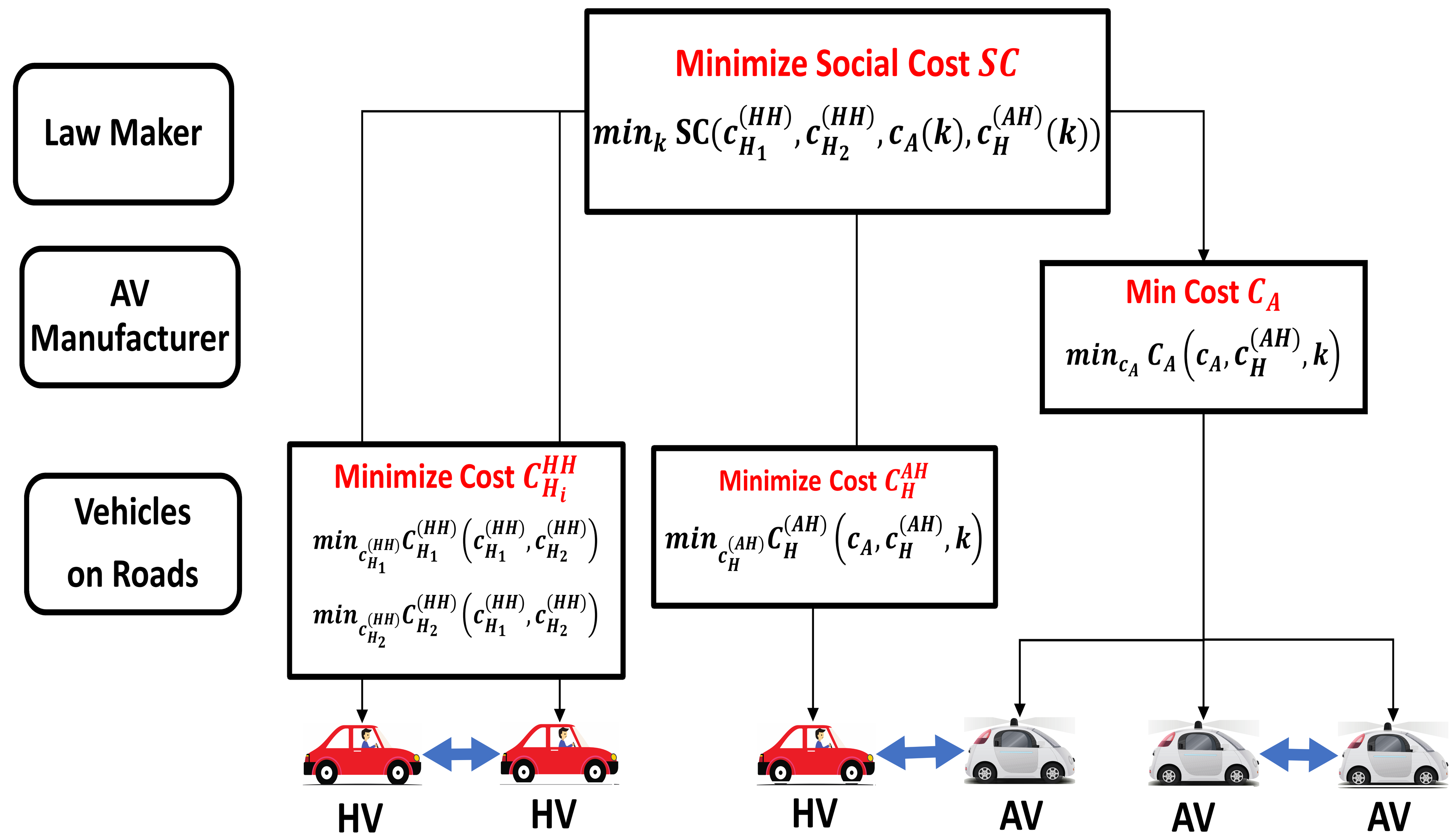
Hierarchical Structure

$$\text{[GameSum]} \min_{k \in \mathcal{C}_k} \quad SC(c_{H_1}^{(HH)}, c_{H_2}^{(HH)}, c_A(k), c_H^{(AH)}(k)), \ 0 < p < 1$$

$$\text{s.t. [GameHH]} \quad c_{H_i}^{(HH)} \in \arg\min_{c_{H_i}^{(HH)} \in \mathcal{C}_{c_H}} \ C_{H_i}^{(HH)}(c_{H_1}^{(HH)}, c_{H_2}^{(HH)}), i = 1, 2$$

$$\text{[GameAH]} \quad c_A \in \arg\min_{c_A \in \mathcal{C}_{c_A}} \ C_A(c_A, c_H^{(AH)}, k)$$

$$\text{s.t.} \quad c_H^{(AH)} \in \arg\min_{c_H^{(AH)} \in \mathcal{C}_{c_H}} \ C_H^{(AH)}(c_H^{(AH)}, c_A, k)$$



Game Equilibrium:

$$c^* = (c_{H_1}^{*(HH)}, c_{H_2}^{*(HH)}, c_A^*(k^*, p), c_H^{*(AH)}(k^*, p), k^*(p))$$

Equilibrium condition:

$$SC(c_{H_1}^{*(HH)}, c_{H_2}^{*(HH)}, c_A^*, c_H^{*(AH)}) \le SC(c_{H_1}^{(HH)}, c_{H_2}^{(HH)}, c_A, c_H^{(AH)})$$

$$C_A(c_A^*, c_H^{*(AH)}, k) \le C_A(c_A, c_H^{*(AH)}, k)$$

$$C_H^{(AH)}(c_H^{*(AH)}, c_A, k) \le C_H^{(AH)}(c_H^{(AH)}, c_A, k)$$

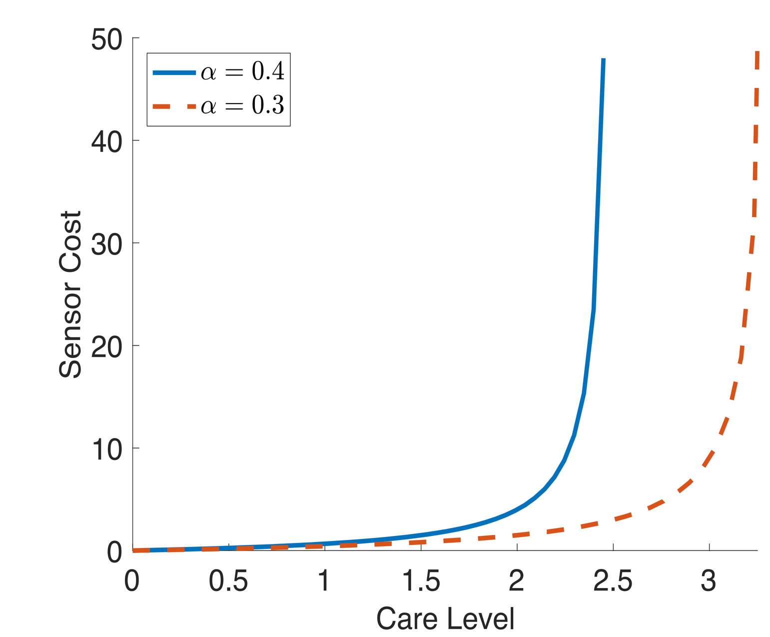
$$C_{H_i}^{(HH)}(c_{H_1}^{*(HH)}, c_{H_2}^{*(HH)}) \le C_{H_i}^{(HH)}(c_{H_1}^{(HH)}, c_{H_2}^{(HH)}), \ i = 1, 2$$

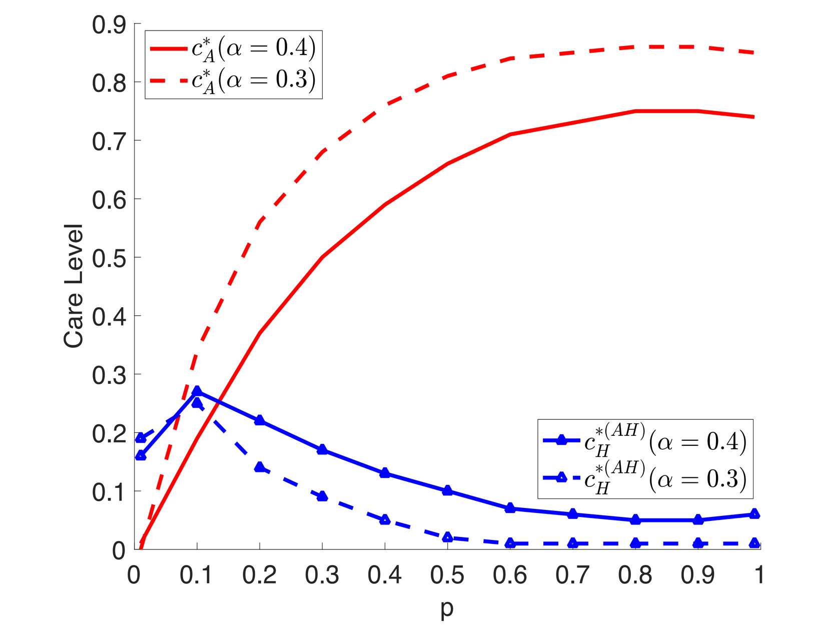
Uniqueness and Existence

In the tri-level game, sufficient conditions of existence and uniqueness of game equilibrium $(c_{H_1}^{*(HH)}, c_{H_2}^{*(HH)}, c_H^{*(AH)}, c_A^*, k^*)$ are:

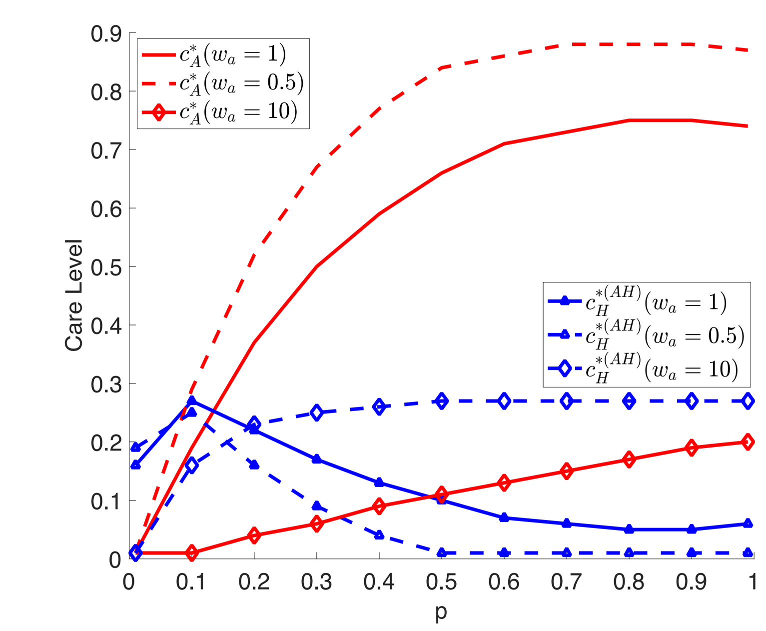
1. $\frac{\partial^2 C_H^{(AH)}(c_A, c_H^{(AH)}, k)}{\partial (c_H^{(AH)})^2} > 0.$
2. $\frac{\partial^2 C_A(c_A, m_{sl}(c_A), k)}{\partial c_A^2} > 0$ where $c_H^{(AH)} \equiv m_{sl}(c_A)$ is the solution to $\frac{\partial C_H^{(AH)}(c_A, c_H^{(AH)}, k)}{\partial c_H^{(AH)}} = 0.$
3. $\frac{d^2 SC(m_l(k), m_{sl}(m_l(k)))}{dk^2} > 0$ where $c_A \equiv m_l(k)$ is the solution to $\frac{\partial C_A(c_A, m_{sl}(c_A), k)}{\partial c_A} = 0.$
4. $J + J^T$ is positive definite, where $J = \begin{bmatrix} \frac{\partial^2 C_{H_1}^{(HH)}(c_{H_1}^{(HH)}, c_{H_2}^{(HH)})}{\partial c_{H_1}^{2(HH)}} & \frac{\partial^2 C_{H_1}^{(HH)}(c_{H_1}^{(HH)}, c_{H_2}^{(HH)})}{\partial c_{H_1}^{(HH)} \partial c_{H_2}^{(HH)}} \\ \frac{\partial^2 C_{H_2}^{(HH)}(c_{H_1}^{(HH)}, c_{H_2}^{(HH)})}{\partial c_{H_1}^{(HH)} \partial c_{H_2}^{(HH)}} & \frac{\partial^2 C_{H_2}^{(HH)}(c_{H_1}^{(HH)}, c_{H_2}^{(HH)})}{\partial c_{H_2}^{2(HH)}} \end{bmatrix}.$

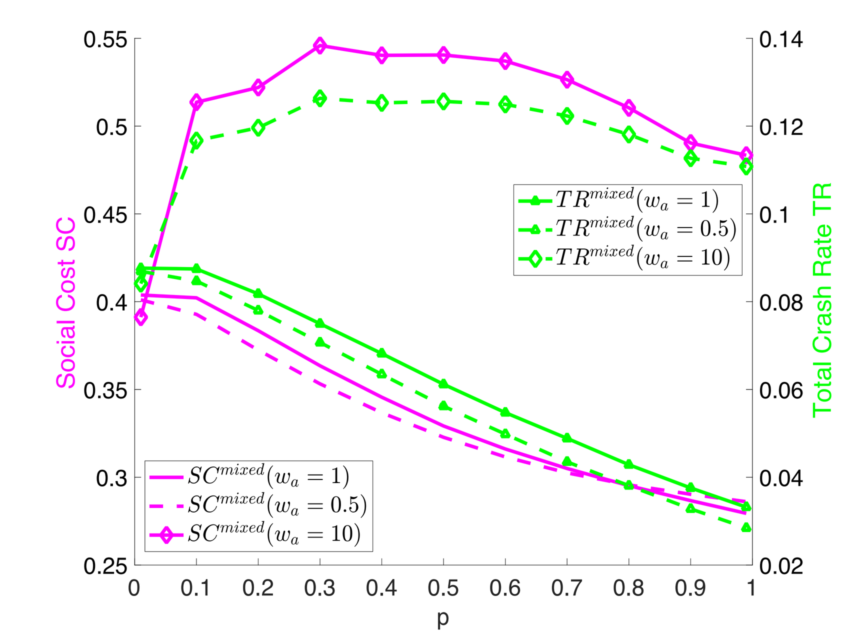
Sensitivity Analysis



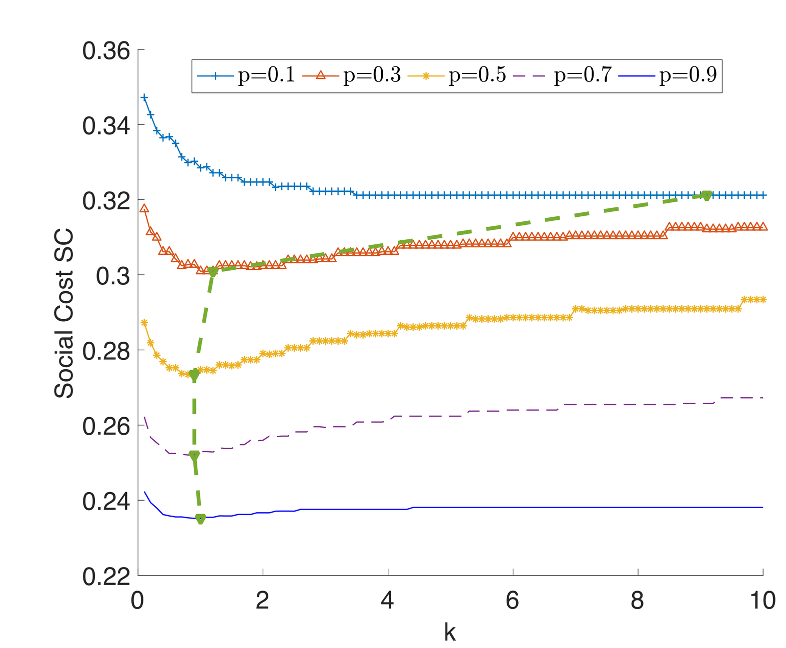


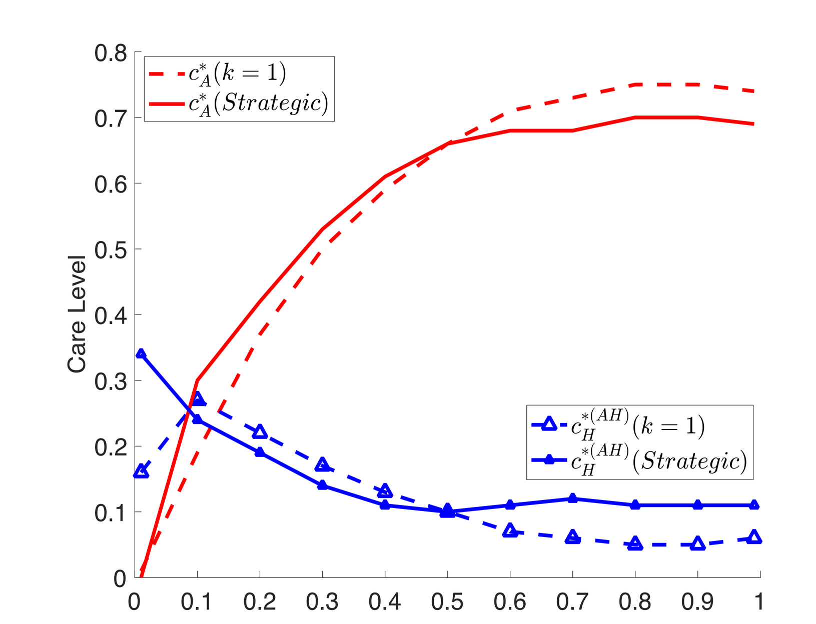
Government subsidies to AV manufacturers for the reduction of production costs would greatly encourage manufacturers to produce AVs that outperform human drivers substantially and improve overall traffic safety and efficiency.





If AV manufacturers are not regulated in terms of AV technology specifications (w_a=10), AV manufacturers tend to be purely profit-oriented and destructive to the overall traffic system.





An optimally designed liability policy is critical to help prevent human drivers from developing moral hazard and to assist the AV manufacturer with a tradeoff between traffic safety and production costs.

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